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ANGLE BASED COMPUTATION WITH CUT-OFF ALGORITHM FOR MOVING TARGET

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ABSTRACT

Performance evaluations of a maneuvering target require Radars, Electro-optical tracking systems and telemetry systems to track the target. Angle based tracking system uses only Azimuth & Elevation which may be generated either from Electro-optical systems or any other antenna based tracking systems which provide Azimuth & Elevation. In such case minimum of any two systems are required for position calculation. For calculation of position, if only two such systems are considered then computation is simple. Our proposed algorithm considers two or more number of tracking systems. For such angle based tracking system, even if the measured azimuth and elevation are within the error limit of the instrument, still the problem of deviation of computed position from the true position occurs as the target moves far away from the systems. This is because of low apex angle generated by tracking systems. In this paper, algorithm to compute the position of the target is presented. Further an algorithm of Cut-off has been presented based on the computed apex angle in real time environment which is used efficiently to eliminate the problem of deviation. Radar measures Azimuth, Elevation & Range information and Telemetry system uses Inertial Navigation System for providing more accurate position information at far distance of the target. The performances of algorithms are analyzed based on various real time track data which are compared with Radar and Telemetry systems.

KEYWORDS: Apex Angle, Electro, Optical Systems, WGS-84 System, Direction Cosines

I. INTRODUCTION

For an airborne target, position calculation is essential with respect to a particular reference point. This reference point may be the starting point of the target or any point towards which the target is moving. Radar based tracking system provides Azimuth, Elevation and Range information of the target so that a single Radar can provide position information of the target with respect to its own position. Then a suitable co-ordinate transformation method is used to compute the position with respect to the reference point. In case of angle based tracking system, a single system can provide only azimuth and elevation information which is insufficient to find the position of the target. In such case, minimum two systems are required for position calculation [1] [2]. Section II presents the algorithm for computing the position of the target for such angle based tracking systems. Section III deals with the analysis of positional deviation with the other sensors data such as Radar and Telemetry. In section IV the algorithm for apex angle computation and cut-off is presented along with the performance analysis of proposed apex based cut off algorithm.

II. COMPUTATION ALGORITHM

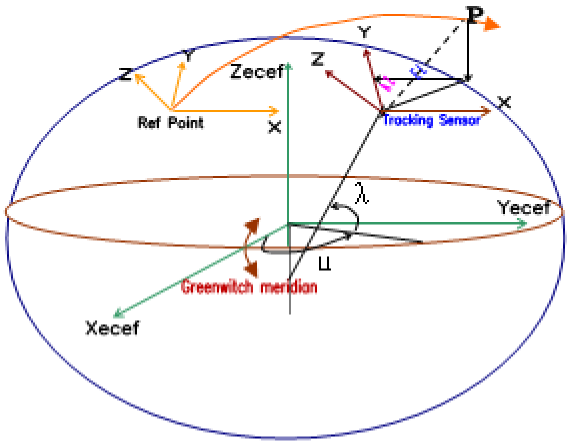

Figure 1: Axis Convention ECEF and ENV

Two frames of reference are used as in Figure 1.

- ECEF (Earth Centered Earth Fixed) axis with respect to centre of the earth and
- Local i.e ENV (East(x) North(y) Vertical (z) axis with respect to any tracking systems or point of reference consider N no of tracking systems each of which provide two parameters
- Azimuth (AZ): The angle measured w.r.t. north horizontally in clockwise direction (0 deg to 360 deg).
- Elevation (EL): Elevation is the angle measured vertically upward (+ve) & downward (-ve) w.r.t. Horizontal plane (XY Plane).

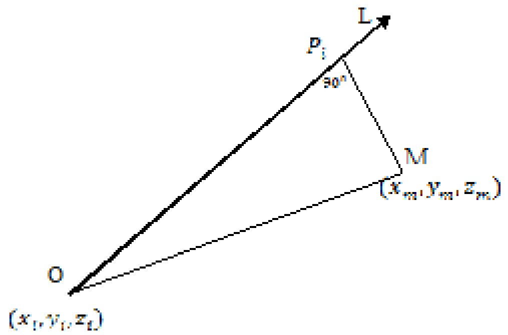

Figure 2: Position of a Tracking Station with its Line of Sight OL

In Figure2, $x_i, y_i z_i$ is the position of ith tracking station w.r.t. reference point's local axis which can be obtained from the following steps.

The coordinate of any point above the earth w.r.t. ECEF axis is [3]

X_{ecef}= (γ+h)cosλcosμ

Y_{ecef}= (γ +h)cosλsinμ

Z_{ecef}= {(1-e^2) γ +h}sinλ

Where λ, μ & h are geodetic latitude, longitude and height respectively of the point above the earth and $\gamma=a/(1-e^2\sin^2\lambda)^{1/2}$ is the effective radius of the earth with the WGS-84 model [4] having the following values

a=6378137.0 ('a' is major axis)

e^2=f (2-f) ('e' is eccentricity)

f=1/298.257223563 ('f' is flattening)

Using the above equation we can obtain the coordinate of ith tracking station: ($X_{stn,}$, Y_{stn} ,$Z_{stn,)}$and reference point ($X_{ref,}$ Y_{ref}, $Z_{ref,)}$ both w.r.t. ECEF axis of reference.

Thus the coordinate of ith tracking station ($x_i, y_{,i} z_i$) wrt reference point's local axis is

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} -\sin\mu & \cos\mu & 0 \\ -\sin\lambda\cos\mu & -\sin\lambda\sin\mu & \cos\lambda \\ \cos\lambda\cos\mu & \cos\lambda\sin\mu & \sin\lambda \end{bmatrix} * \begin{bmatrix} x_{stn} - x_{ref} \\ y_{stn} \quad -y_{ref} \\ z_{stn} \quad -z_{ref} \end{bmatrix} \tag{1}$$

Matrix-1

Where matrix-1 is the ECEF to Local alignment matrix and λ, μ are the geodetic latitude and longitude of the reference point [5].

In Figure 2, x_m, y_m, z_m is the unknown position of target M, w.r.t. reference point's local axis which is to be found out from following steps . OL is the line of sight with which ith tracking station looking to target which may not exactly point towards the target M due to it's measurement error.

Let $l_i^{'}$, $m_i^{'}$& $n_i^{'}$ be the direction cosines of line OP_i , where these direction cosines are w.r.t. local axis of ith tracking station at point O which is computed from its measured Azimuth(AZ) and Elevation(EL) values as follows.

$l_i^{'}$=cosE$l_i$sinAZ_i

$m_i^{'}$=cosE$l_i$cosAZ_i

$n_i^{'}$=sinEl_i

Now new direction cosines of the same line OP_i parallel to local axis of common reference point is

$$\begin{bmatrix} l_i \\ m_i \\ n_i \end{bmatrix} = \begin{bmatrix} -\sin\mu & \cos\mu & 0 \\ -\sin\lambda\cos\mu & -\sin\lambda\sin\mu & \cos\lambda \\ \cos\lambda\cos\mu & \cos\lambda\sin\mu & \sin\lambda \end{bmatrix} * \begin{bmatrix} -\sin\mu & -\sin\lambda\cos\mu & \cos\lambda\cos\mu \\ \cos\mu & -\sin\lambda\sin\mu & \cos\lambda\sin\mu \\ 0 & \cos\lambda & \sin\lambda \end{bmatrix} * \begin{bmatrix} l_i^{'} \\ m_i^{'} \\ n_i^{'} \end{bmatrix}$$

Matrix-2 **Matrix-3**

where Matrix-3 is Local to ECEF alignment matrix of ith tracking station in which case λ and μ are the Geodetic Latitude and Longitude of ith tracking station respectively and Matrix-2 is the ECEF to Local alignment matrix of reference point in which case λ and μ are the Geodetic Latitude and Longitude of reference point respectively [4].

Projection of OM on OL is OP_i (Figure 2)

$OP_i = (x_m - x_i)\, l_i + (y_m - y_i)\, m_i + (z_m - z_i)\, n_i$

NOW $MP_i^2 = OM^2 - OP_i^2 = (x_m - x_i)^2 + (y_m - y_i)^2 + (z_m - z_i)^2 - \{(x_m - x_i)\, l_i + (y_m - y_i)\, m_i + (z_m - z_i) n_i\}^2$

Sum of the square of perpendicular distances contributed by total N tracking stations is:

$$S = \sum_{i=1}^{N} (MP_i)^2 = \sum_{i=1}^{N} ((x_m - x_i)^2 - (y_m - y_i)^2 - (z_m - z_i)^2) - \sum_{i=1}^{N} \{(x_m - x_i)\, l_i + (y_m - y_i)\, m_i + (z_m - z_i) n_i\}^2 \tag{2}$$

The above sum S is minimum when $\frac{\partial s}{\partial x_m}$ =0, $\frac{\partial s}{\partial y_m}$ =0 and $\frac{\partial s}{\partial z_m}$ =0[6]. Thus from above equation (2) $\frac{\partial s}{\partial x_m}$ = 0 gives the following equation.

$$x_m (N - \sum_{i=1}^{N} l_i^2) - y_m \sum_{i=1}^{N} l_i m_i - z_m \sum_{i=1}^{N} l_i n_i = \sum_{i=1}^{N} x_i (1 - l_i^2) - \sum_{i=1}^{N} y_i\, l_i m_i - \sum_{i=1}^{N} z_i\, l_i n_i \quad (3)$$

Similarly $\frac{\partial s}{\partial y_m}$ = 0 gives the following equation (4)

$$-x_m \sum_{i=1}^{N} l_i m_i + y_m (N - \sum_{i=1}^{N} m_i^2) - z_m \sum_{i=1}^{N} m_i n_i = \sum_{i=1}^{N} y_i (1 - m_i^2) - \sum_{i=1}^{N} z_i\, m_i n_i - \sum_{i=1}^{N} x_i\, l_i m_i \quad (4)$$

and $\frac{\partial s}{\partial z_m}$ = 0 gives the following equation (5)

$$-x_m \sum_{i=1}^{N} l_i n_i - y_m \sum_{i=1}^{N} m_i n_i + z_m (N - \sum_{i=1}^{N} n_i^2) = \sum_{i=1}^{N} z_i (1 - n_i^2) - \sum_{i=1}^{N} x_i\, l_i n_i - \sum_{i=1}^{N} y_i\, m_i n_i \quad (6)$$

Writing equations 3, 4 & 5 in matrix form, we get the following matrix equation.

$$\begin{bmatrix} N - \sum_{i=1}^{N} l_i^2 & -\sum_{i=1}^{N} l_i m_i & -\sum_{i=1}^{N} l_i n_i \\ -\sum_{i=1}^{N} l_i m_i & N - \sum_{i=1}^{N} m_i^2 & -\sum_{i=1}^{N} m_i n_i \\ -\sum_{i=1}^{N} l_i n_i & -\sum_{i=1}^{N} m_i n_i & N - \sum_{i=1}^{N} n_i^2 \end{bmatrix} * \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} x_i (1 - l_i^2) - \sum_{i=1}^{N} y_i l_i m_i - \sum_{i=1}^{N} z_i l_i n_i \\ \sum_{i=1}^{N} y_i (1 - m_i^2) - \sum_{i=1}^{N} z_i m_i n_i - \sum_{i=1}^{N} x_i l_i m_i \\ \sum_{i=1}^{N} z_i (1 - n_i^2) - \sum_{i=1}^{N} x_i l_i n_i - \sum_{i=1}^{N} y_i m_i n_i \end{bmatrix}$$

Thus solving above matrix equation gives the unknown position $x_{m,}$ $y_{m,}$ & z_m of the target w.r.t. reference point.

III. DEVIATION ANALYSIS

It is observed that in case of long range tracking, due to small measurement error in both azimuth and elevation of each tracking station, deviation in position measurement occurs as computed by above algorithm. This deviation is very small and it is within the error limit if the target distance is comparatively less with respect to the measuring stations. As the distance of the target increases from tracking stations, this deviation increases due to measurement error [7]. In some other cases also, where the target is not far away from the measuring stations, deviation is also observed in some particular pattern of trajectory path of the target. In both cases the main reason of deviation is due to the geographical position of the flight vehicle with respect to measuring stations which is finally attributed to the low apex angle. The deviations as observed for both of the above cases will be explained subsequently.

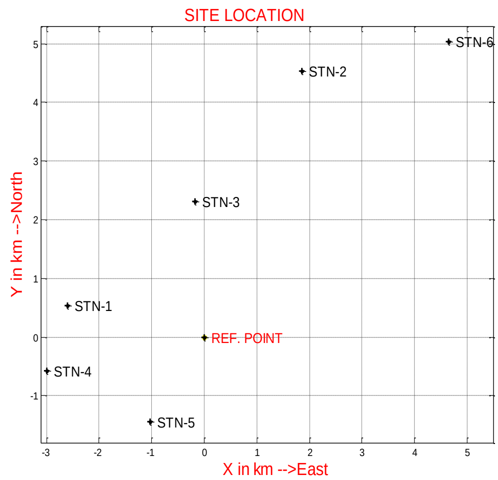

Figure 3: Location of Six Different Tracking Station in XY Plane

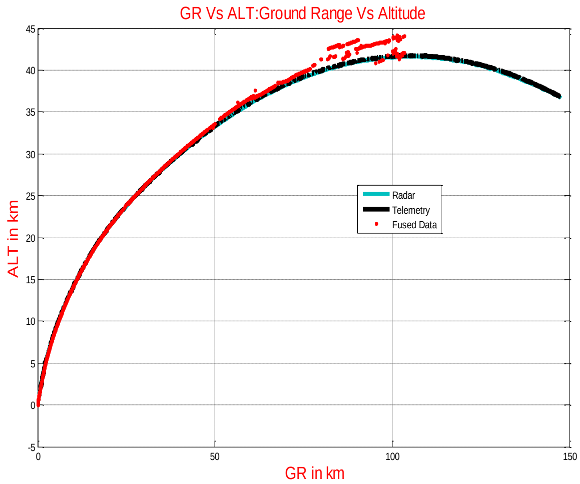

Figure 4: Example of Unacceptable GR vs ALT Plot with Deviation before Cutoff

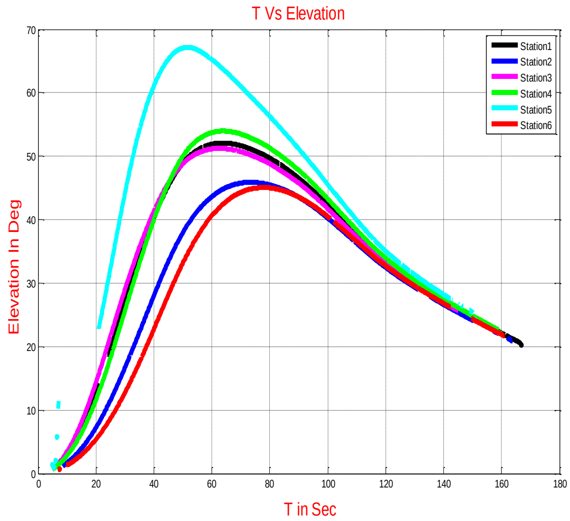

Figure 5: Time vs Elevation Plot of Six Tracking Stations

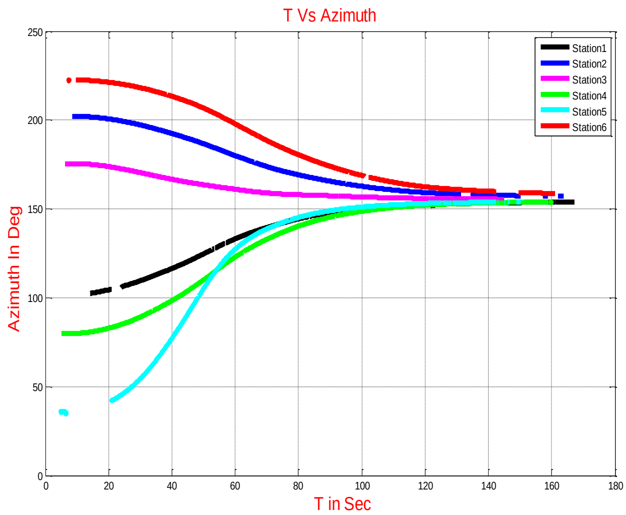

Figure 6: Time vs Azimuth Plot of Six Tracking Stations

Figure 3 represents site locations of six tracking stations given in XY plane w.r.t. reference point which are located within 10 km from the reference point. The target moved from the reference point attending height of more than 40 km and ground range more than 145 km w.r.t. reference point whose track is given by radar, telemetry & fused track generated by six

tracking stations using the above computational algorithm in sec-II. The track result in Ground Range verses Altitude is shown in Figure 4. The elevation and azimuth values received from each of the tracking station for the corresponding track are shown in Figure 5 and Figure 6 respectively along the time axis in which few gaps in the plot represent track loss by the stations. In this case when the target is moving away from all tracking stations, it is natural that all azimuth and elevations will appear to converge and hence the apex angle between any two stations will gradually decrease which will cause deviation of the computed position as compared to radar and telemetry for small error in measurement in each tracking station [8] [9]. This deviation is clear from the Figure 4.

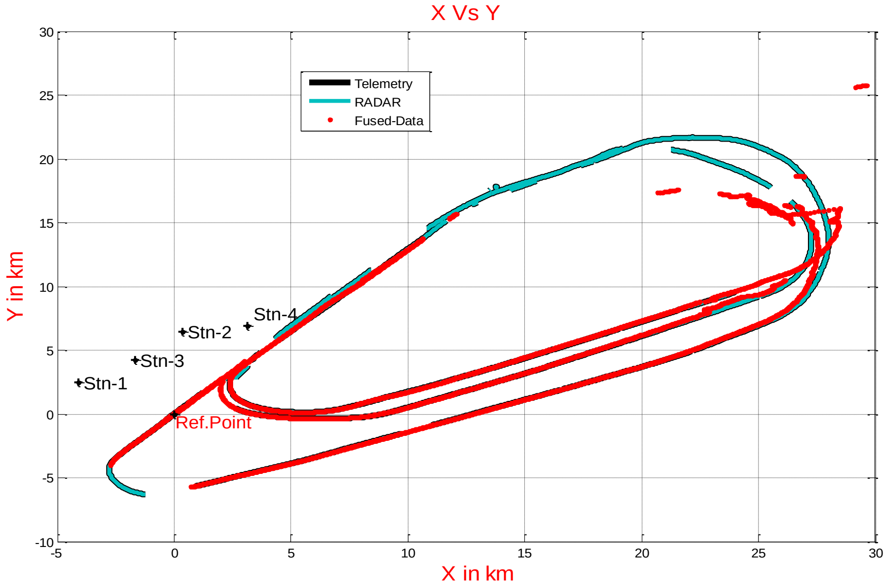

Figure 7: Example of Unacceptable X vs Y Plot with Deviation before Cutoff

Figure 7 shows another example, in which the track result of target is shown in xy plane. In this case the target moved around the four tracking stations and reference point where the height of the target was between 400 meter and 2.5 km. The new positions of four tracking stations along with reference point is also shown in Figure 7 The elevation and azimuth values received from each of the tracking stations are shown in Figure 8 and Figure 9 respectively along the time axis. It is seen in Figure 7 that the fused data deviated from radar and telemetry data in particular region where the apex angle due to four tracking stations appears to be less.

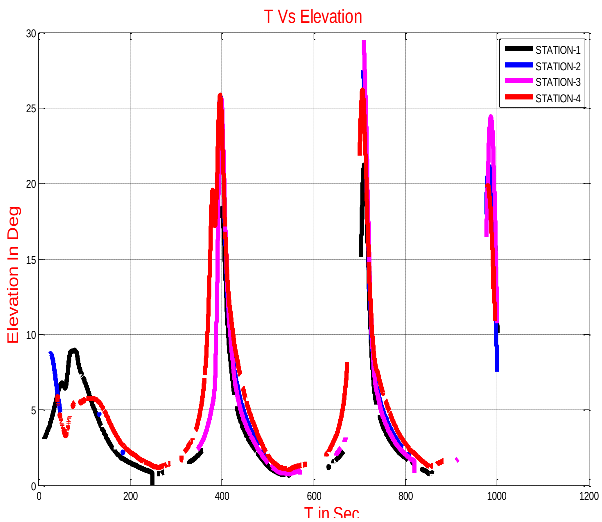

Figure 8: Time vs Elevation Plot of Four Tracking Stations

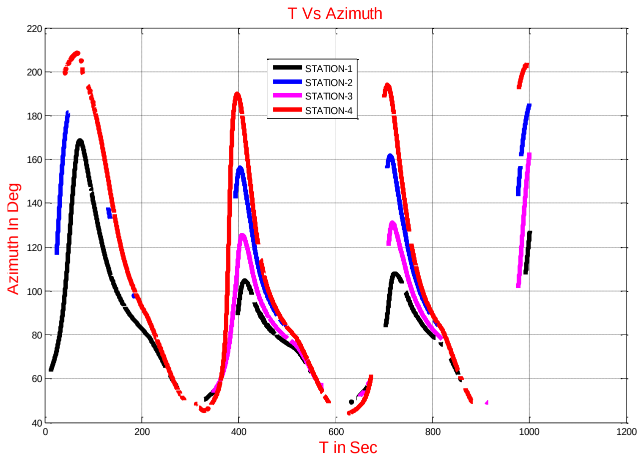

Figure 9: Time vs Azimuth Plot of Four Tracking Stations

IV. ALGORITHM FOR APEX ANGLE COMPUTATION AND CUT-OFF

Figure 10 shows positions of any two tracking stations and target in space where θ is the apex angle between the two tracking stations. To avoid the deviations of the track result as shown in Figure 4 and Figure 7, it is necessary to apply cut-off to the computed data (Fused Data) based on apex angle between pair of tracking stations. Following steps represent the algorithm for apex angle computation and cut-off.

Step1: Prior to real time track, compute the square of distances between stations and put in the Matrix-4 (assumed size: 6×6) where position of each station is known w.r.t. reference point as computed in equation (1) of section –II based on Lat, Long & Alt values of each station. Let D_{ij}^2 be the square of distance from i_{th} to j_{th} station, where $1 \leq i < N$ & $i + 1 \leq j \leq N$, N=total no of stations.

Since the measurement in this case is distance, D_{ji}^2 becomes redundant once D_{ij}^2 is computed and hence the Matrix-4 becomes upper triangular as shown.

$$\begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Matrix-4

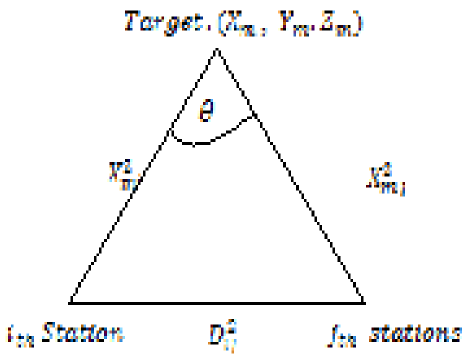

Figure 10: Apex Angle θ between two Tracking Stations

Step 2: In real time, get the position of target X_m, Y_m, Z_m from the existing algorithm mentioned in section II

Step 3: For N tracking stations, get at most N values of distances and its square between target & tracking station if each station is able to track the target at the moment, otherwise make these values zero. Let X_{mi} , X_{mi}^2 be these values for i'th station.

Step 4: Apply the following formula for apex angle between two valid i'th & j'th tracking station using the law of cosines for triangle, where θ is the apex angle. $\text{Cos}\ \theta = \frac{x_{mi}^2 + x_{mj}^2 - D_{ij}^2}{2 * x_{mi} * x_{mj}}$

Where $1 \le i < N$ & $i + 1 \le j \le N$ & θ is apex angle between i_{th} station and j_{th} station.

Step 5: Quit from the above loop (step 2 to 4) if $\mathbf{8^0 \le \boldsymbol{apex} \le 172^0}$ between any two stations and consider the data (X_m, Y_m, Z_m) to be valid, otherwise continue the loop to search for any combinations of two stations giving apex such that $\mathbf{8^0 \le \boldsymbol{apex} \le 172^0}$. If all the combinations finish without this condition then computed position of target to be considered as invalid due to low apex angle.

The complexity of the above cutoff algorithm is O (n^2).

The above apex based cut-off algorithm was applied to the track data of six tracking stations whose fused data was represented in Figure 4 and the result obtained is shown as in Figure 11, in which the rest of the fused data matched with the telemetry and radar data. Similarly Figure 12 was obtained when the same algorithm was applied to the data of four tracking stations whose fused data was represented in Figure 7.

Thus in both the cases the deviated data was removed in real time by the above apex based cutoff algorithm, resulting the matching of fused data with radar and telemetry in real time environment.

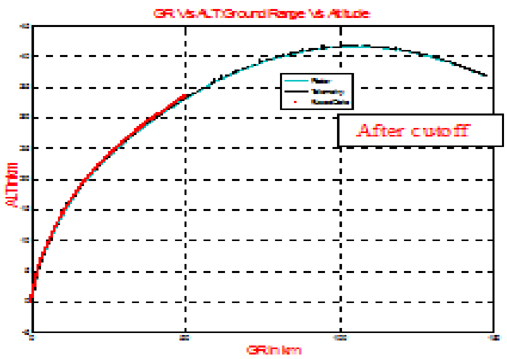

Figure 11: Acceptable GR vs ALT Plot after Cutoff

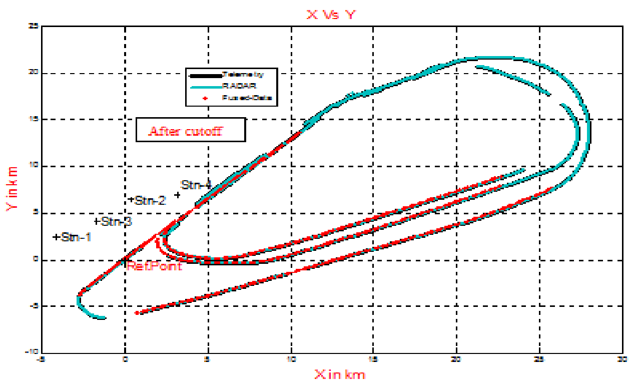

Figure 12: Acceptable X vs Y Plot after Cutoff

V. CONCLUSIONS

Algorithm proposed in section–II can compute accurate target positions throughout the complete trajectory path of the target for the theoretical accurate angular measurement of each tracking station. But practically this is not possible due to inherent error in angular measurement of each tracking station, and hence the deviation was more prominent outside the apex window. Cut off implementation based on apex requires computation of apex angle by considering pairs of tracking stations which needs efficient algorithm for processing in real time from various possible combinations. The analysis was done for two cases such that in first case the target moved away from the reference point and tracking station with continuously increasing height. In this case deviation resulted in the ground range verses height plot as in Fig.4. In the second case target moved around the reference point and tracking station in which case deviation resulted in XY plane as shown in Figure 7. In both the cases the above cutoff algorithm could able to remove the deviated data there by retaining the track data that was comparable up to the accuracy of 98.9% with ther sensors data such as radars and telemetry. Apex window: $\mathbf{8^0 \leq \mathit{apex} \leq 172^0}$ was fixed based on track results of various type of trajectories followed by target where as in each case it could give satisfactory result. Both lower limit (8^0) and upper limit (172^0) ensure any of the two tracking station's line of sight do not appear to become parallel, in which case there will be large deviation of computed position for small error in measurement. However further analysis is required to achieve the above apex window based on performance of the particular tracking sensors and position accuracy needed in a particular application.

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